(1)

تم التصحيح رياضيات (٤) طاقة ⓵ فصل ثاني ١٨-١٩

السؤال الأول (5) (1)

$$(z+1)^4 + z^4 = 0 \Rightarrow$$

$$\left(\frac{z+1}{z}\right)^4 = -1 \qquad \frac{z+1}{z} = (-1)^{\frac{1}{4}} \Rightarrow \frac{z+1}{z} = e^{i\left(\frac{\pi}{4}+\frac{\pi k}{2}\right)}$$

$$z+1 = z\, e^{i\left(\frac{\pi}{4}+\frac{\pi k}{2}\right)} \Rightarrow z\left(1 - e^{i\left(\frac{\pi}{4}+\frac{\pi k}{2}\right)}\right) = -1$$

$$z_k = -\frac{1}{1-e^{i\left(\frac{\pi}{4}+\frac{\pi k}{2}\right)}} = -\frac{1-e^{-i\left(\frac{\pi}{4}+\frac{\pi k}{2}\right)}}{\left|1-e^{i\left(\frac{\pi}{4}+\frac{\pi k}{2}\right)}\right|}$$

$$z_k = -\frac{1-\left[\cos\left(\frac{\pi}{4}+\frac{\pi k}{2}\right) - i\sin\left(\frac{\pi}{4}+\frac{\pi k}{2}\right)\right]}{\left|1-\cos\left(\frac{\pi}{4}+\frac{\pi k}{2}\right) - i\sin\left(\frac{\pi}{4}+\frac{\pi k}{2}\right)\right|}$$

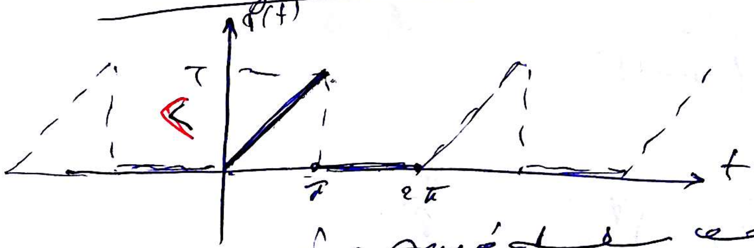
$$z_k = -\frac{1-\cos\left(\frac{\pi}{4}+\frac{\pi k}{2}\right) - i\sin\left(\frac{\pi}{4}+\frac{\pi k}{2}\right)}{\left(1-\cos\left(\frac{\pi}{4}+\frac{\pi k}{2}\right)\right)^2 + \sin^2\left(\frac{\pi}{4}+\frac{\pi k}{2}\right)}$$

$$z_k = -\frac{1-\cos\left(\frac{\pi}{4}+\frac{\pi k}{2}\right) - i\sin\left(\frac{\pi}{4}+\frac{\pi k}{2}\right)}{2\left(1-\cos\left(\frac{\pi}{4}+\frac{\pi k}{2}\right)\right)}$$

$$z_k = -\frac{1}{2} + \frac{i}{2}\,\frac{\sin\left(\frac{\pi}{4}+\frac{\pi k}{2}\right)}{1-\cos\left(\frac{\pi}{4}+\frac{\pi k}{2}\right)} \Rightarrow Re(z_k) = -\frac{1}{2}$$

تم تصحيح الرياضيات (٤) قسم الطاقة سنة ثانية ٢٠١٨-٢٠١٩ فصل ثاني

السؤال الثاني (ب) (20)



التابع دوري دوره 2π غير زوجي ولا فردي لذلك نشره حسب سلسلة فورييه عامة

$$f(t) = a_0 + \sum_{1}^{\infty} a_n\cos nt + b_n\sin nt$$

$$a_0 = \frac{1}{2\pi}\left[\int_0^{\pi} t\,dt + \int_{\pi}^{2\pi} 0\,dt\right] = \boxed{\frac{\pi}{4}}$$

$$a_n = \frac{2}{2\pi}\left[\int_0^{\pi} t\cos nt\,dt + \int_{\pi}^{2\pi} 0\cos nt\,dt\right]$$

$$a_n = \frac{1}{\pi}\left\{\frac{t}{n}\sin nt + \frac{1}{n^2}\cos nt\right\}_0^{\pi}$$

$$= \frac{1}{\pi}\left\{\frac{\pi}{n}(0) + \frac{1}{n^2}\left[(-1)^n - 1\right]\right\}$$

$$= \frac{1}{\pi n^2}\left[(-1)^n - 1\right]$$

$$b_n = \frac{2}{2\pi}\left\{\int_0^{\pi} t \sin nt\, dt + \int_{\pi}^{2\pi} 0 \, dt\right\}$$

$$= \frac{1}{\pi}\left\{-\frac{t}{n}\cos nt + \frac{1}{n^2}\sin nt\right\}_0^{\pi}$$

$$= \frac{-1}{\pi}\left\{\frac{\pi}{n}(-1)^n\right\} = -\frac{1}{n}(-1)^n$$

$$f(t) = \frac{\pi}{4} + \sum_1^{\infty} \frac{1}{n^2}\left((-1)^n - 1\right)\cos nt - \frac{1}{n}(-1)^n \sin nt$$

بالإبدال ب $t=0$ والترتيب

$$-\frac{\pi^2}{4} = \sum_1^{\infty} \frac{1}{n^2}\left\{(-1)^n - 1\right\}$$

$$-\frac{\pi^2}{4} = -2\sum_1^{\infty} \frac{1}{(2n-1)^2} \Rightarrow$$

$$\sum_1^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$$

(٣)

(أ) السؤال الثاني ١٧

(١٧)

$$I = \int_0^{\frac{\pi}{2}} \cot^{\frac{1}{2}}\theta \, d\theta$$

$$I = \int_0^{\frac{\pi}{2}} \cos^{\frac{1}{2}}\theta \, \sin^{-\frac{1}{2}}\theta \, d\theta$$

٤

$$= \int_0^{\frac{\pi}{2}} \cos^{2n-1}\theta \, \sin^{2m-1}\theta \, d\theta$$ بافتراض

٢

$$2n-1 = +\frac{1}{2} \qquad n = \frac{3}{4}$$ ٣

$$2m-1 = -\frac{1}{2} \qquad m = \frac{1}{4}$$ ٣

$$I = \frac{1}{2}\beta\left(\frac{1}{4}, \frac{3}{4}\right) = \frac{1}{2}\,\frac{\Gamma(\frac{1}{4})\Gamma(\frac{3}{4})}{\Gamma(1)}$$

٢

$$= \frac{1}{2}\,\frac{\frac{\pi}{\sin \pi/4}}{1} = \frac{1}{2}\,\frac{\pi}{\frac{\sqrt{2}}{2}} = \frac{\pi}{\sqrt{2}}$$ ٣

(ب) انشر الدالة $f(z) = \frac{1}{\sinh z - z}$ في جوار $z = 0$

(١٣) ١٣

$$f(z) = \frac{1}{\left(z + \frac{z^3}{3!} + \frac{z^5}{5!} - \right) - z} = \frac{1}{\frac{z^3}{3!} + \frac{z^5}{5!} - }$$

٤

$$= \frac{1}{\frac{z^3}{3!}\left\{1 + \frac{3!}{5!}z^2 + \frac{3!}{7!}z^4 + \frac{3!}{9!}z^6 - \right.}$$

٤

$$= \frac{3!}{z^3}\,\frac{1}{1+t} = \frac{3!}{z^3}\left(1 - t + t^2 + t^3 - \right.$$

٤

$$= \frac{3!}{z^3}\left\{1 - \left(\frac{3!}{5!}z^2 + \frac{3!}{7!}z^4 - \right) + \left(\frac{3!}{5!}z^2 + \frac{3!}{7!}z^4 - \right)^2 - - \right.$$

(ع)

$$= \frac{3!}{z^3}\left\{1 - \frac{3!}{5!}z^2 + O(z^4) - - -\right\}$$

$$= \frac{3!}{z^2} - \frac{(3!)^2}{5!}\frac{1}{z} + O(z)$$

فالنقطة الشاذة

$z=0$ قطب من رتبة ثالثة وقيمة الاستنباط

$$Res(0) = -\frac{(3!)^2}{5!}$$

$$Res(s) = -\frac{36}{120} = -\frac{3}{10}$$

$$I = 2\pi i\,(Res(0)) = -\frac{2\pi i\, 3}{10} = -\frac{3\pi i}{5}$$

السؤال الثالث (١٢)

$$y'' + y = \cos(t-1)u(t-1) \qquad y_0 = y'_0 = 0$$

بأخذ لابلاس للطرفين

$$\mathcal{L}y'' + \mathcal{L}y = \mathcal{L}(\cos(t-1)u(t-1))$$

$$s^2Y - sy_0 - y'_0 + Y = e^{-s}\frac{s}{s^2+1}$$

$$Y(s^2+1) = e^{-s}\frac{s}{s^2+1}$$

$$Y = e^{-s}\frac{s}{(s^2+1)^2}$$

$$y = \mathcal{L}^{-1}\frac{s}{(s^2+1)^2}\Big|_{t \to t-1}$$

(5)

$$\mathcal{L}^{-1}\frac{s}{(s^2+1)^2} = \frac{1}{s^2+1}\,\frac{s}{s^2+1}$$

$F_1(s) = \frac{1}{s^2+1} \qquad f(t) = \sin t$

$F_2(s) = \frac{s}{s^2+1} \qquad f_1(t) = \cos t$

$$f(t) = \int_0^t \sin\tau\cos(t-\tau)\,d\tau$$

$$= \frac{1}{2}\int_0^t [\sin t + \sin(2\tau - t)]\,d\tau$$

$$= \frac{1}{2}\left[\tau\sin t - \frac{1}{2}\cos(2\tau - t)\right]_0^t$$

$$= \frac{1}{2}\left(t\sin t - \frac{1}{2}\cos t + \frac{1}{2}\cos t\right)$$

$$= \frac{1}{2}t\sin t$$

$$y = \mathcal{L}^{-1}Y = \frac{1}{2}(t\sin t)u(t)\Big|_{t \to t-1}$$

$$y = \frac{1}{2}[(t-1)\sin(t-1)u(t-1)]$$

(٦)

$$\frac{\partial^2 u}{\partial t^2} = \frac{1}{4}\frac{\partial^2 u}{\partial x^2}$$

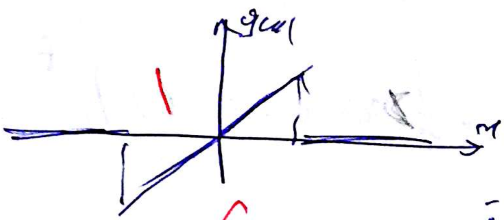
$$g(x) = \begin{cases} 2x & -1 < x < 1 \\ 0 & \text{عدا ذلك} \end{cases}$$

نحسب الإزاحة الابتدائي معدوم نطبق حل المعادلة

$$u(x,t) = \frac{1}{2a}\left[G(x+at) - G(x-at)\right]$$

حيث $G(x) = \int g(u)\,du$ ، $a = \frac{1}{2}$

بحساب $G(x)$ حيث $g(x)$ هي الدالة السابقة

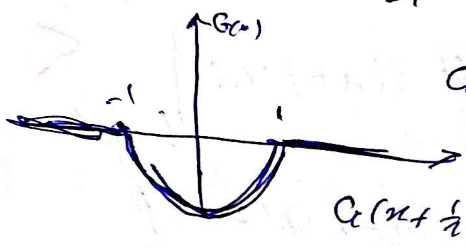


$$G(x) = \int_{-\infty}^{x} 0\,du = 0 \qquad x < -1$$

$$G(x) = \int_{-\infty}^{-1} 0\,du + \int_{-1}^{x} 2u\,du \qquad x < 1$$

$$G(x) = x^2 - 1$$

$$G(x) = \int_{-\infty}^{-1} 0\,du + \int_{-1}^{1} u\,du + \int_{1}^{x} 0\,du = 0 \qquad x > 1$$



$$G(x) = (x^2 - 1)\left[U(x+1) - U(x-1)\right]$$

$$G(x + \tfrac{1}{2}t) = \left[(x + \tfrac{1}{2}t)^2 - 1\right]\left[U(x + \tfrac{1}{2}t + 1) - U(x + \tfrac{1}{2}t - 1)\right]$$

$$G(x - \tfrac{1}{2}t) = \left[(x - \tfrac{1}{2}t)^2 - 1\right]\left[U(x - \tfrac{1}{2}t + 1) - U(x - \tfrac{1}{2}t - 1)\right]$$

$$u(x,t) = G(x + \tfrac{1}{2}t) - G(x - \tfrac{1}{2}t)$$